UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF ELECTRONIC ENGINEERING

MAIN EXAMINATION 2006

TITLE OF PAPER	**:**	**MATHEMATICAL METHODS I (PAPER ONE)**
COURSE NUMBER	**:**	**E370(I)**
TIME ALLOWED	**:**	**THREE HOURS**
INSTRUCTIONS	**:**	**ANSWER <u>ANY FOUR</u> OUT OF FIVE QUESTIONS.** **EACH QUESTION CARRIES <u>25</u> MARKS.** **MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.**

THIS PAPER HAS <u>SEVEN</u> PAGES, INCLUDING THIS PAGE.

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Question one

Given the following inhomogeneous second order ordinary differential equation as :

$$2\frac{d^2 f(t)}{dt^2} + 5\frac{d f(t)}{dt} + 13 f(t) = 20\cos(2t) - 15\sin(2t)$$

(a) set the particular solution of $f(t)$ as $k_1 \sin(7t) + k_2 \cos(7t)$, find the values of k_1 *and* k_2 and thus the particular solution of $f(t)$, namely $f_p(t)$,

(8 marks)

(b) find the general solution of the homogeneous part of the given equation , i.e.,

$$2\frac{d^2 f(t)}{dt^2} + 5\frac{d f(t)}{dt} + 13 f(t) = 0$$, and name it as $f_h(t)$, (5 marks)

(c) (i) write down the general solution of the above inhomogeneous equation in terms of $f_p(t)$ and $f_h(t)$, and name it as $f_g(t)$, (2 marks)

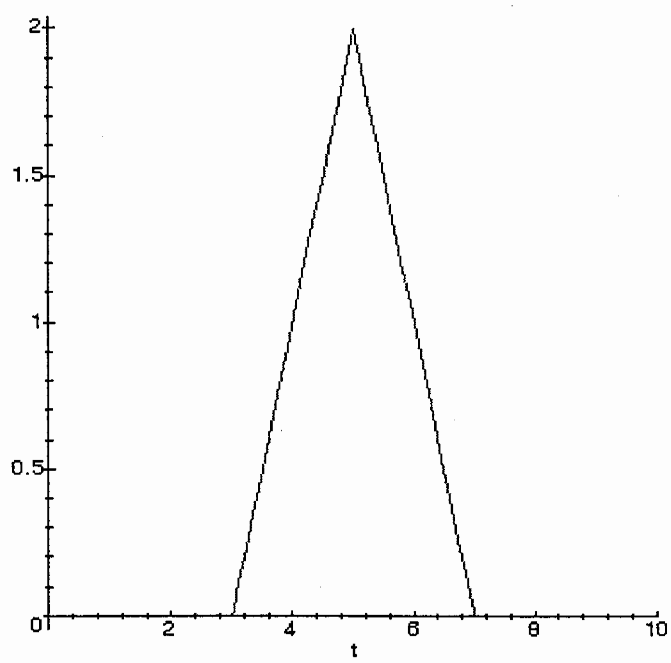
(ii) if the initial conditions are given as $f(0) = 6$ and $f'(0) = 4$, determine the values of the arbitrary constants in $f_g(t)$ and thus the specific solution of $f(t)$, name it as $f_s(t)$. Plot both $f_s(t)$ and $f_p(t)$ for $t = 0$ *to* 15 , and show them in a single display. Compare their behaviour at large t and make a brief remark. (10 marks)

Question two

Given the following inhomogeneous second order ordinary differential equation as :

$$\frac{d^2 f(t)}{dt^2} + 3\frac{d f(t)}{dt} + 9 f(t) = g(t)$$

(a) (i) if $g(t)$ is a pulse function and is given as follows :



(i.e., $g(t) = 0 \quad for \quad t \le 3 \quad and \quad t \ge 7$ and the peak value of $g(t)$ is 2 happened at $t = 5$)

write down the above pulse function of t in terms of Heaviside functions and plot it for $t = 0 \; to \; 10$ to reproduce the above diagram. (5 marks)

(ii) find the Laplace transform of $g(t)$ given in (a) (i) and named it as $G(s)$.

(2 marks)

(b) (i) find, $F(s)$, the Laplace transform of $f(t)$ if $f(0) = 4$ and $f'(0) = 3$.

Show that $F(s)$ can be rewritten as $F(s) = K(s) + H(s)\,G(s)$

where $G(s)$ is obtained in (a)(ii). Find the expressions of $K(s)$ and

$H(s)$. (7 marks)

(ii) find the inverse Laplace transform of $K(s)$ and $H(s)$, and name them

as $k(t)$ and $h(t)$ respectively, (3 marks)

(iii) find the convolution of $h(t)$ and $g(t)$, and name it as $hg(t)$,

(5 marks)

(iv) write down the specific solution of $f(t)$ in terms of $k(t)$ and $hg(t)$

and plot it for $t = 0$ *to* 10. (3 marks)

Question three

(a) Given the system of linear equations in matrix form as $A\,X = b$ where

$$A = \begin{pmatrix} 6 & -3 & 8 \\ 5 & -2 & -9 \\ 2 & -5 & -4 \end{pmatrix}, \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad b = \begin{pmatrix} -13 \\ 7 \\ 9 \end{pmatrix},$$

(i) *augment* A *and* b, then apply the Gauss elimination method using commands of *addrow* *and* *backsub* to find the solution of X.

(5 marks)

(ii) use the Cramer's rule to find the solution X. Compare the answer obtained here with that obtained in (a)(i). (5 marks)

(b) Given the following system of differential equations for coupled oscillators as :

$$\begin{cases} \dfrac{d^2 x_1(t)}{d t^2} = -\,18\, x_1(t) + 15\, x_2(t) \\ \dfrac{d^2 x_2(t)}{d t^2} = 3\; x_1(t) - 6\; x_2(t) \end{cases}$$

(i) set $x_1(t) = X_1\, e^{i\omega t}$ and $x_2(t) = X_2\, e^{i\omega t}$, deduce the following matrix equation $-\,\omega^2 \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = A \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$ where $A = \begin{pmatrix} -18 & 15 \\ 3 & -6 \end{pmatrix}$

(4 marks)

(ii) find the eigenvalues and eigenvectors of A and thus evaluate the eigenfrequencies ω, (6 marks)

(iii) find the normal coordinates of the system. (5 marks)

Question four

Given the following differential equation :

$$\frac{d^2 y(x)}{d x^2} - 5\frac{d\,y(x)}{d\,x} + 4\,y(x) = 0$$

(a) set $y(x) = \sum_{n=0}^{\infty} a_n x^{n+s}$ *and* $a_0 \neq 0$, use the power series method to find the indicial equations and solve for the values of s and also the expression of a_1 in terms of s *and* a_0 , (7 marks)

(b) find the recurrence relation, (3 marks)

(c) for each values of s found in (a) , set $a_0 = 1$ and find the values of a_1 , a_2 , a_3, ... , a_{10} by using the second indicial equation in (a) and the recurrence relation in (b). Then write down two particular solutions expressed in power series and truncated to the a_{10} term. (10 marks)

(d) use *dsolve* command to find the two particular solutions and then express them into power series. Compare these power series with those obtained in (c) and make a brief remark. (5 marks)

Question five

(a) Given the following ensemble of data S representing student marks as :

[60,30,87,59,63,53,80,34,53,84,66,64,55,64,46,46]

(i) find the values of mean , variance and standard deviation of S (5 marks)

(ii) use the interval of 5 , starting from 29.5 and ending at 89.5 , i.e.,

$(29.5, 34.5), (34.5, 39.5), \cdots\cdots$, to plot a histogram of S (8 marks)

(b) Use the random number generator in MAPLE to generate an ensemble S of 30 data values ranging from 0 *to* 100 , and then find its mean value . (4 marks)

(c) For a normal distribution $f(x)$ with the mean value of 15 and the standard deviation of 5 ,

(i) plot $f(x)$ for $x = 0$ *to* 30 , (3 marks)

(ii) find its corresponding cumulative distribution function $g(x)$ and use it to calculate the values of the probabilities of $P(x > 9)$ and $P(3 < x < 20)$.

(5 marks)